Diffusion in biased turbulence

M. Vlad,[1] F. Spineanu,[1] J. H. Misguich,[2] and R. Balescu[3]
[1]Association Euratom–NASTI Romania, National Institute for Laser, Plasma and Radiation Physics, P.O. Box MG-36, Magurele, Bucharest, Romania
[2]Association Euratom–CEA sur la Fusion, CEA–Cadarache, F-13108 Saint-Paul-lez-Durance, France
[3]Association Euratom–Etat Belge sur la Fusion, Université Libre de Bruxelles, Boulevard du Triomphe, 1050 Bruxelles, Belgium
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Particle transport in two-dimensional divergence-free stochastic velocity fields with constant average is studied. Analytical expressions for the Lagrangian velocity correlation and for the time-dependent diffusion coefficients are obtained. They apply to stationary and homogeneous Gaussian velocity fields.

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I. INTRODUCTION

This paper deals with the diffusion processes induced at large scales in stochastic velocity fields. More precisely, we consider particle motion in two-dimensional incompressible velocity fields that can be either static or time-dependent stochastic fields. In this context, the effects of a constant average value of the stochastic velocity on particle diffusion is determined. This is a generic problem that appears in various studies in fluid and plasma turbulence, astrophysics, meteorology, oceanography, or solid state physics [1].

The diffusion coefficient and the mean square displacement (MSD) depend on the Lagrangian velocity correlation (LVC), a rather complicated statistical quantity that contains the stochastic velocity field and the resulting stochastic trajectories. A dimensionless quantity, the Kubo number K, is defined (see Sec. II) in order to measure the capacity of the trajectories to explore the space structure of the stochastic velocity field before the latter changes due to the time variation. At small values of K (quasilinear regime) the time variation of the stochastic velocity is fast and the trajectories have no time to ''see'' the shape of this field. The results concerning the statistics of the trajectories are well established in this case: the displacements are Gaussian and have a mean square growing linearly in time with a diffusion coefficient $D_{ql} \sim K^2$ [2,3]. At large K (nonlinear regime), the direct numerical simulations [4–6] have shown that a dynamical trapping of the trajectories appears and produces the modification of the statistical properties of the displacements [4,7]. They are non-Gaussian and the diffusion coefficient scales as $D_{tr} \sim K^\gamma$ with $\gamma = 0.7$ for the spectrum considered in [6]. The existing analytical methods completely fail in describing this trapping process [4] and the studies usually rely on the renormalization-group techniques [8,9] or on qualitative estimates [10], and consequently they determine only the asymptotic diffusion coefficient or the asymptotic behavior of the MSD. We have proposed in [11] and [12] a statistical approach that determines the LVC and the time-dependent diffusion coefficients. Analytical results valid for an arbitrary value of the Kubo number are obtained for stochastic velocity fields with a stationary and homogeneous Gaussian distribution, with a zero average, and with a given Eulerian correlation function. The method of decorrelation trajectories [11] is extended here to include stochastic velocity fields that have a constant average. The LVC and the time-dependent (running) diffusion coefficient in a biased turbulence are thus determined.

We show that the presence of an average component of the velocity $\mathbf{V}_d$ produces the following effects. In a static velocity field $\mathbf{v}(\mathbf{x})$, the drift $\mathbf{V}_d$ determines a transition from a subdiffusive to a superdiffusive or diffusive process, depending on the ratio of V_d to the amplitude V of the stochastic velocity. At small values of $\mathbf{V}_d$ ($V_d \ll V$), the diffusion coefficient along $\mathbf{V}_d$ does not saturate but it is linear in time. The MSD is superdiffusive of ballistic type and scales as t^2. The average displacement is linear in time but the average Lagrangian velocity is smaller than V_d. We show that these are nonlinear effects determined by trajectory trapping in the structure of the stochastic velocity field. When $V_d \gg V$, a finite diffusion coefficient in the direction of $\mathbf{V}_d$ is obtained and the average Lagrangian velocity is equal to the Eulerian average V_d. The transport across $\mathbf{V}_d$ remains subdiffusive for all values of V_d.

In time-dependent stochastic velocity fields with nonzero average, both the diffusion coefficients along and across $\mathbf{V}_d$ are finite. The average Lagrangian velocity equals $\mathbf{V}_d$. This behavior is determined by the time variation of the stochastic velocity field which eventually releases all trajectories. We show that the process of trajectory trapping combined with the average drift determines an ''anomalous'' diffusion regime with a strongly increased diffusion coefficient along $\mathbf{V}_d$ and a significantly reduced perpendicular diffusion. A detailed study of the possible diffusion regimes is presented.

The problem of average drifts was studied before for the quasilinear case [13–16] or by means of an analogy with percolation processes [17,10,18]. The effect of an average velocity is also treated in the context of diffusion advection or random walks (see the review paper [8]) but these results cannot be compared to ours since particle collisions considered in these models change significantly the effective diffusion coefficients. We do not attempt here to evaluate the probability distribution function of particle displacements as reviewed in a very recent paper [19] but restrict at determining the first two moments. Our results are qualitatively similar with those obtained numerically in [20] and [21] where features as trajectory trapping, enhanced diffusion along the average drift, and ballistic regimes are evidenced in particle motion in the two-dimensional turbulence with large vortical structures generated by the Hasegawa-Wakatani or Hasegawa-Mima equations.

The paper is organized as follows. The formulation of the generic problem is presented in Sec. II together with a short summary of the results obtained in the absence of $\mathbf{V}_d$. The solution for the static case is presented in Sec. III. The diffusion in a time-dependent biased stochastic stream function is studied in Sec. IV. The conclusions are summarized in Sec. V.

II. TURBULENT DIFFUSION PROBLEM

The test particle and passive scalar turbulent diffusion problem or the diffusion induced by continuous movements relies on the Langevin equation

$$\frac{d\mathbf{x}(t)}{dt}=\mathbf{v}(\mathbf{x}(t),t), \quad \mathbf{x}(0)=\mathbf{0} \tag{1}$$

where $\mathbf{v}(\mathbf{x},t)$ is a stochastic velocity field. The Langevin equation (1) describes the motion of some point particles which are advected by the stochastic velocity field. We consider here a two-dimensional space where $\mathbf{x}(t)$ represents the trajectory of the particle in Cartesian coordinates $\mathbf{x}\equiv(x_1,x_2)$. The stochastic velocity $\mathbf{v}(\mathbf{x},t)$ is a divergence-free stochastic field: $\boldsymbol{\nabla}\cdot v(\mathbf{x},t)=0$ and it can thus be determined from a stream function $\phi(\mathbf{x},t)\mathbf{e}_z$, as

$$\mathbf{v}(\mathbf{x},t)=\nabla\times\phi(\mathbf{x},t)\mathbf{e}_z, \tag{2}$$

where $\mathbf{e}_z$ is the unitary vector of the z axis. In studies of magnetically confined plasmas, the velocity in Eq. (1) is the $\mathbf{E}\times\mathbf{B}$ drift, $\mathbf{v}(\mathbf{x},t)=-\boldsymbol{\nabla}\phi^e\times\mathbf{e}_z/B$, where $\phi^e(\mathbf{x},t)$ is the electrostatic potential and B is the magnetic field strength considered to be constant. Thus the definition of the velocity field in plasma turbulence is similar with Eq. (2) with $\phi=-\phi^e/B$. In both cases the two components v_1 an v_2 of the velocity are determined in terms of a scalar stochastic field $\phi(\mathbf{x},t)$ which is considered here to be a stationary and homogeneous Gaussian field. The distribution of the stream function is thus determined by the average $\Phi(\mathbf{x},t)\equiv\langle\phi(\mathbf{x},t)\rangle$ and by the two-point Eulerian correlation (EC) function $E(\mathbf{x},t)$. The latter is modeled by

$$E(\mathbf{x},t)\equiv\langle\tilde{\phi}(\mathbf{x}_1,t_1)\tilde{\phi}(\mathbf{x}_1+\mathbf{x},t_1+t)\rangle=\beta^2\mathcal{E}(|\mathbf{x}|)h(|t|), \tag{3}$$

where $\tilde{\phi}(\mathbf{x},t)\equiv\phi(\mathbf{x},t)-\langle\phi(\mathbf{x},t)\rangle$ is the fluctuating part of $\phi(\mathbf{x},t)$. Angular brackets denote the statistical average over the realizations of the stochastic stream function and β is the amplitude of the fluctuations of ϕ. $\mathcal{E}(\mathbf{x})$ is a dimensionless function having a maximum at $\mathbf{x}=\mathbf{0}$, where its value is $\mathcal{E}(\mathbf{0})=1$, and which tends to zero as $|\mathbf{x}|\to\infty$. It actually depends on the dimensionless variable $\mathbf{x}/\lambda$, where λ is the correlation length. $h(t)$ is a dimensionless, decreasing function of time varying from $h(0)=1$ to $h(\infty)=0$. It depends on the ratio t/τ_c, where τ_c is the correlation time. A dimensionless parameter, the *Kubo number* can be defined as

$$K=V\tau_c/\lambda, \quad V=\beta/\lambda, \tag{4}$$

where V measures the amplitude of the fluctuating velocity. The Kubo number is thus the ratio of the average distance covered by the particles during the correlation time of the stochastic velocity field to its correlation length. It represents a measure of the particle's capacity of exploring the space structure of the velocity field before the latter changes. In mathematical terms, this Kubo number determines the importance of the Lagrangian nonlinearity introduced in Eq. (1) by the space dependence of the velocity field.

Since the velocity components are derivatives of the potential, they are Gaussian, stationary, and homogeneous as well. The two-point EC of the velocity components, $E_{ij}(\mathbf{x},t)\equiv\langle\tilde{v}_i(\mathbf{0},0)\tilde{v}_j(\mathbf{x},t)\rangle$, and of the potential with the velocity, $E_{\phi i}\equiv\langle\tilde{\phi}(\mathbf{0},0)\tilde{v}_i(\mathbf{x},t)\rangle$, are obtained from $E(\mathbf{x},t)$ by the appropriate derivatives,

$$E_{11}=-\frac{\partial^2}{\partial x_2^2}E, \quad E_{22}=-\frac{\partial^2}{\partial x_1^2}E, \quad E_{12}=\frac{\partial^2}{\partial x_1\partial x_2}E,$$

$$E_{1\phi}=-E_{\phi 1}=-\frac{\partial}{\partial x_2}E, \quad E_{2\phi}=-E_{\phi 2}=\frac{\partial}{\partial x_1}E. \tag{5}$$

We assume that the velocity has a constant average value (which is chosen to be along x_1 axis)

$$\langle\mathbf{v}(\mathbf{x},t)\rangle=V_d\mathbf{e}_1 \tag{6}$$

and consequently the average stream function is

$$\Phi(\mathbf{x},t)=V_d x_2. \tag{7}$$

The mean square displacement (MSD) of the particles and the running diffusion coefficient are determined from the two-point correlation function of the Lagrangian velocity (LVC). The latter is defined as

$$L_{ij}(t)\equiv\langle\tilde{v}_i(\mathbf{x}(0),0)\tilde{v}_j(\mathbf{x}(t),t)\rangle, \tag{8}$$

where $\tilde{\mathbf{v}}(\mathbf{x}(t),t)=\mathbf{v}(\mathbf{x}(t),t)-\langle\mathbf{v}(\mathbf{x}(t),t)\rangle$ is the fluctuating part of the velocity along the trajectory (i.e., of the Lagrangian velocity). The MSD can be written as

$$\langle x_i^2(t)\rangle=2\int_0^t d\,\tau L_{ii}(\tau)(t-\tau) \tag{9}$$

and the running diffusion coefficient, defined as $D_{ii}(t)\equiv\frac{1}{2}(d/dt)\langle x_i^2(t)\rangle$, is

$$D_{ii}(t)=\int_0^t d\tau\, L_{ii}(\tau), \tag{10}$$

provided that the LVC is stationary. The aim of this paper is to determine the LVC, knowing the statistical description of the stochastic stream function.

For small Kubo numbers (quasilinear regime), the results are well established [13–16]: the diffusion coefficient is $D_{QL}=(\lambda^2/\tau_c)K^2$ in the absence of an average drift ($V_d=0$) and this value remains practically unchanged for small drift velocities, $V_d \ll \lambda/\tau_c$. At large values of V_d ($V_d \gg \lambda/\tau_c$), the diffusion coefficient along $\mathbf{V}_d$ becomes

$$D_{11} \sim V^2 \lambda / V_d = (\lambda^2/\tau_c) K V / V_d. \tag{11}$$

Thus, it decreases with V_d as V_d^{-1} and is independent of τ_c. The diffusion coefficient perpendicular to $\mathbf{V}_d$ is as well a decreasing function of V_d but it depends on the space shape of the EC of the stream function,

$$D_{22} \sim \frac{1}{V_d} \left| \mathcal{E}' \left(\frac{V_d \tau_c}{\lambda} \right) \right|. \tag{12}$$

These estimates also hold at large Kubo numbers if V_d is larger than the amplitude of the stochastic velocity V. Thus, the general condition for the regime (11),(12) is $V_d \gg \max(\lambda/\tau_c, V)$.

At large K, in the absence of the drift velocity ($V_d = 0$), due to the slow time variation, the trajectories can follow approximately the contour lines of $\phi(\mathbf{x},t)$. The space structure of the stochastic stream function has an important influence on particle trajectories. This produces a trapping effect: the trajectories are confined for long periods in small regions. A typical trajectory shows an alternation of large displacements and trapping events. The latter appear when the particles are close to the maxima or minima of the potential and consists of trajectory winding (for many turns) on almost closed small size paths. The large displacements are produced when the trajectories are at small absolute values of the potential. We have developed in Ref. [11] a statistical method that succeeds in describing this trapping process. It shows that the asymptotic diffusion coefficient has an algebraic dependence on K, $D_{tr} \sim (\lambda^2/\tau_c) K^\gamma$ with a value of $\gamma = 0.62$ obtained there for a particular EC of the stream function. Generally, the exponent γ slightly depends on the large $|\mathbf{x}|$ asymptotic behavior of the function $\mathcal{E}(|\mathbf{x}|)$ in the Eulerian correlation of $\phi(\mathbf{x},t)$ and varies around the above value. In the limit case of static stream functions (frozen turbulence) corresponding to $\tau_c = \infty$, $K = \infty$, the trapping is permanent and consequently particle motion is subdiffusive. The MSD is still a growing function of time $\langle x_i^2(t) \rangle \sim t^\gamma$ due to the large size contour lines of $\phi(\mathbf{x})$ that allow large displacements. The probability density for the displacements is determined in [22] where a non-Gaussian result is found due to the memory effects induced by the long time correlation of the Lagrangian velocity.

A finite average velocity ($V_d \neq 0$) can strongly influence these results. The effect of V_d is studied in the next section for the static stream function and in Sec. IV for the time-dependent turbulence.

III. FROZEN TURBULENCE

We consider first the case of static stream functions $\phi(\mathbf{x})$ that correspond to $\tau_c = \infty$ or $K = \infty$. In these conditions the EC of the stream function depends only on the distance $|\mathbf{x}|$ and $h(t) = 1$ in Eq. (3). We use dimensionless quantities (without changing the notations) with the following units: λ for distances, β for the stream function, and $V = \beta/\lambda$ for the velocities. The unit of time is the characteristic flight time of the particles through the correlation length of the stochastic field: $\tau_f = \lambda/V$. In the following calculations, V_d is thus the ratio of the average velocity to the amplitude V of the fluctuating velocity. The Langevin equation (1) can be written in dimensionless quantities as

$$\frac{d\mathbf{x}(t)}{dt} = \tilde{\mathbf{v}}(\mathbf{x}(t)) + \mathbf{V}_d, \quad \mathbf{x}(0) = \mathbf{0}. \tag{13}$$

We use the method presented in [11] for determining the LVC and the diffusion coefficient for the trajectories obtained from Eq. (13). Actually, we apply here the space-time decorrelation method presented in the Appendix of Ref. [11]. The same method was used in Ref. [12] for determining the effect of particle collisions on the diffusion in stochastic velocity fields. This method is able to describe the complex process of diffusion and intrinsic trapping in the structure of the stochastic field. We show here that the success of this approach is due to its property of reproducing the invariance of the Lagrangian stream function.

The essential point of our method consists of solving the Langevin equation (13) in (disjoint) subsets of the ensemble of realizations of the stochastic stream function. These subensembles (S) are characterized by given values of the stream function and of the velocity at the starting point of the trajectories $\mathbf{x} = \mathbf{0}$, $t = 0$,

$$\phi(\mathbf{0}) = \phi^0, \quad \mathbf{v}(\mathbf{0}) = \mathbf{v}^0. \tag{14}$$

The stream function and the velocity reduced at the subensemble (S) are still Gaussian stochastic fields but they are nonhomogeneous and have modified average values that depend on the EC,

$$\Phi^S(\mathbf{x}) \equiv \langle \phi(\mathbf{x}) \rangle_S = V_d x_2 + \phi^0 E(\mathbf{x}) + (v_j^0 - V_d \delta_{j1}) E_{j\phi}(\mathbf{x}), \tag{15}$$

$$V_i^S(\mathbf{x}) \equiv \langle v_i(\mathbf{x}) \rangle_S = V_d \delta_{i1} + \phi^0 E_{\phi i}(\mathbf{x}) + (v_j^0 - V_d \delta_{j1}) E_{ji}(\mathbf{x}), \tag{16}$$

where $\langle \cdots \rangle_S$ represents the average over the realizations in the subensemble (S). The condition of the zero divergence is reflected in the expressions of the Eulerian average values (15) and (16) which are connected through an equation similar to (2),

$$\mathbf{V}^S(\mathbf{x}) = \left(\frac{\partial}{\partial x_2}, -\frac{\partial}{\partial x_1} \right) \Phi^S(\mathbf{x}), \tag{17}$$

which shows that the average velocity in the subensemble (S) is divergence-free: $\nabla \cdot \mathbf{V}^S(\mathbf{x},t) = 0$.

We note that such subensemble (or conditional) average velocities were used for describing Eulerian properties of stochastic fields because even in a homogeneous turbulence they exhibit interesting structures [23]. Studies of the subensemble Lagrangian averages are presented in [24] and [25]; they are different from the present analysis.

Since the stream function is conserved along the trajectory in each realization, the average Lagrangian stream function in the subensemble (S) is

$$\langle \phi(\mathbf{x}(t))\rangle_S=\phi^0 \tag{18}$$

at any time. A deterministic trajectory $\mathbf{X}(t;S)$ can be defined in each subensemble (S) such that the average of the Eulerian stream function in (S) [Eq. (15)] calculated along this trajectory equals the average Lagrangian stream function (18),

$$\Phi^S(\mathbf{X}(t;S))=\langle \phi(\mathbf{x}(t))\rangle_S=\phi^0. \tag{19}$$

This ''decorrelation trajectory'' can be obtained from a Hamiltonian system of equations with $\Phi^S(\mathbf{X})$ as a Hamiltonian function,

$$\frac{d\mathbf{X}(t;S)}{dt}=\left(\frac{\partial}{\partial X_2},-\frac{\partial}{\partial X_1}\right)\Phi^S(\mathbf{X}(t;S)) \tag{20}$$

and with the initial condition $\mathbf{X}(0;S)=\mathbf{0}$. The Hamiltonian is conserved along the solution of Eq. (20): $\Phi^S(\mathbf{X}(t;S))=\Phi^S(\mathbf{0})=\phi^0$ and thus $\mathbf{X}(t;S)$ ensures the equality (19). The average Lagrangian velocity in (S) is approximated in Ref. [11] by the corresponding Eulerian quantity calculated along the deterministic trajectory $\mathbf{X}(t;S)$,

$$\langle \mathbf{v}(\mathbf{x}(t))\rangle_S\cong\mathbf{V}^S(\mathbf{X}(t;S)). \tag{21}$$

Since the latter is determined from Eq. (20), where actually the right-hand side is the average Lagrangian velocity, one can deduce that $\mathbf{X}(t;S)$ is an approximation of the average trajectory in (S),

$$\mathbf{X}(t;S)\cong\langle \mathbf{x}(t)\rangle_S. \tag{22}$$

We note that actually the approximation (21) for the average Lagrangian velocity in (S) can be obtained using Corrsin factorization in (S) and neglecting all cumulants of the stochastic trajectories except the first one [1]. It is possible to keep the second cumulant, too, but it has a negative effect on the results: the trapping process is not properly described and the invariance of the average Lagrangian stream function in (S) is lost. It is known [1,2] that when such approximations are performed in the whole set of realizations, the results are even more inaccurate and they do not describe at all the trapping process and the invariance of the stream function. Thus, the space-time decorrelation trajectory method [11] is rather good, although it is apparently based on a rough approximation. Any attempt to improve this method should satisfy the requirement of the invariance of the stream function.

The average Lagrangian quantities in the whole ensemble of realizations of ϕ can be obtained by summing up the contributions of each subensemble (S). The average Lagrangian velocity and the LVC [defined in Eq. (8)] can be written as

$$\langle \mathbf{v}(\mathbf{x}(t))\rangle=\int\int d\phi^0\, d\mathbf{v}^0\, P_1(\phi^0)P_1(\mathbf{v}^0)\langle \mathbf{v}(\mathbf{x}(t))\rangle_S, \tag{23}$$

$$L_{ij}(t)=\int\int d\phi^0\, d\mathbf{v}^0\, P_1(\phi^0)P_1(\mathbf{v}^0)\langle v_i(\mathbf{0})v_j(\mathbf{x}(t))\rangle_S - V_{di}\langle v_j(\mathbf{x}(t))\rangle_S, \tag{24}$$

where $P_1(\phi^0)$ and $P_1(\mathbf{v}^0)$ are, respectively, the Gaussian probability densities of the stream function and of the velocity in the point $\mathbf{x}=\mathbf{0}$. We note that the LVC in the subensemble (S) is not stationary as the LVC in whole set of realizations. However, one needs to estimate only $\langle v_i(\mathbf{0})v_j(\mathbf{x}(t))\rangle_S$ which is simply $\langle v_i(\mathbf{0})v_j(\mathbf{x}(t))\rangle_S=v_i^0\langle v_j(\mathbf{x}(t))\rangle_S$. Thus, the LVC and the average Lagrangian velocity can be obtained by estimating the average Lagrangian velocity in each subensemble (S). The latter is obtained by solving Eqs.(20) and using Eq. (21). The LVC can thus be written as

$$L_{ij}(t)\cong\int\int d\tilde{\phi}^0\, d\tilde{\mathbf{v}}^0\, P_1(\tilde{\phi}^0)P_1(\tilde{\mathbf{v}}^0)\tilde{v}_i^0 V_j^S(\mathbf{X}(t;S)). \tag{25}$$

The time-dependent diffusion coefficient is obtained by integrating Eq. (25) according to Eq.(10) as

$$D_{ij}(t)\cong\int\int d\tilde{\phi}^0\, d\tilde{\mathbf{v}}^0\, P_1(\tilde{\phi}^0)P_1(\tilde{\mathbf{v}}^0)\tilde{v}_i^0 X_j(t;S) \tag{26}$$

and is thus determined by the average trajectories in the subensembles. We note that Eqs. (25) and (26) are approximate equations valid for arbitrary values of the Kubo number and of V_d.

In order to obtain an explicit solution for the LVC and the diffusion coefficient, we choose a model for the Eulerian correlation of the stream function (3):

$$E(r)\equiv\mathcal{E}(r)=\frac{1}{1+\dfrac{r^2}{2}}, \tag{27}$$

where $r\equiv|\mathbf{x}|$. The equations (20) for the average trajectory in (S) become:

$$\frac{dX_1}{dt}=V_d+\phi^0\frac{X_2E'}{R}+\frac{u}{R^3}[(X_1^2E'+X_2^2RE'')\cos\alpha - X_1X_2(RE''-E')\sin\alpha],$$

$$\frac{dX_2}{dt}=-\phi^0\frac{X_1E'}{R}-\frac{u}{R^3}[X_1X_2(RE''-E')\cos\alpha - (X_2^2E'+X_1^2RE'')\sin\alpha], \tag{28}$$

where α is the angle between $\tilde{\mathbf{v}}^0$ and $\mathbf{V}_d$, $u=|\tilde{\mathbf{v}}^0|$, $E'(R)$, $E''(R)$ are the first and second derivatives of $E(R)$, and $R=\sqrt{X_1^2+X_2^2}$. The trajectory obtained from Eq. (28) evolves on the contour line of the average stream function $\Phi^S(\mathbf{X})=\Phi^S(\mathbf{0})=\phi^0$ in the subensemble. The effect of the drift velocity V_d is the opening of a set of paths. This can be seen in

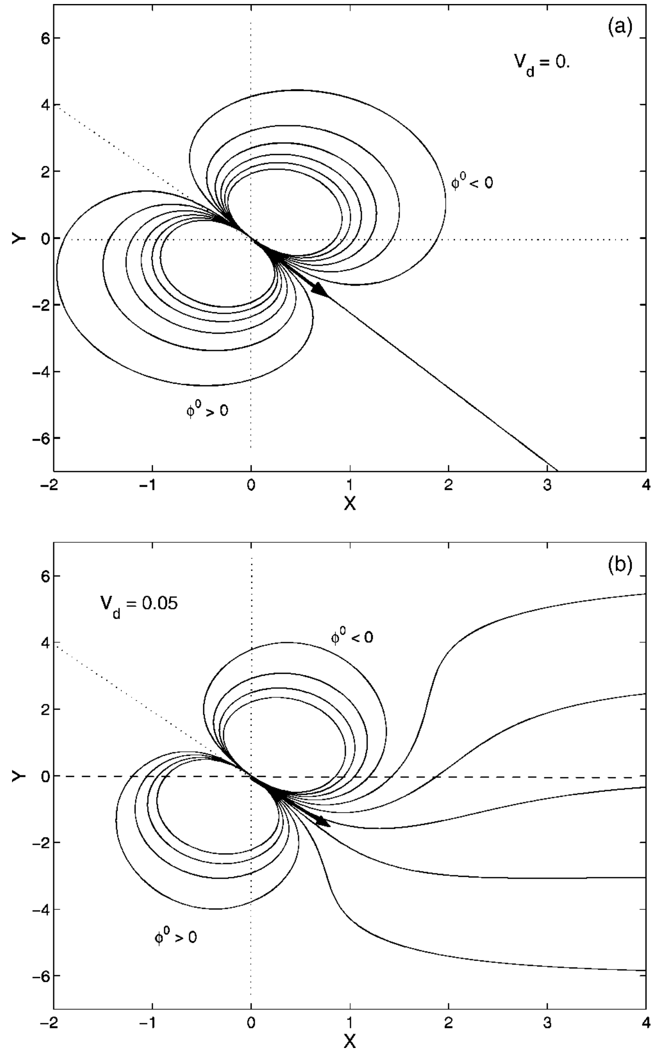

FIG. 1. (a) Examples of average paths for the driftless case $V_d=0$ for $\phi^0=0,\ \pm 0.1,\ \pm 0.2,\ldots,u=1$ and $\alpha=-\pi/4$. The size of the paths decreases continuously with the increase of $|\phi^0|$. (b) Same as in (a), but for a nonzero average velocity, $V_d=0.05$.

Fig. 1 where some paths corresponding to $V_d=0$ and $V_d=0.1$ are plotted. In the driftless case, all paths except that corresponding to $\phi^0=0$ are closed curves [Fig. 1(a)], while at $V_d\neq 0$ the paths are opened for an interval of values of ϕ^0 around $\phi^0=0$ whose size increases when V_d increases [Fig. 1(b)]. There are still trapped average trajectories (in the subensembles with large absolute values of the stream function ϕ^0) but the drift V_d has a releasing effect for the average trajectory in the subensembles with small ϕ^0. The average trajectories obtained from Eq. (28) are functions of time, of the three parameters that define the subensemble (S), ϕ^0, u, α, and of V_d. In the driftless case, the average trajectories are function of a scaled time $\theta=ut$ and of a single parameter $p=\phi^0/u$. The drift velocity V_d breaks the isotropy of the space introducing the dependence on α and also leads to more complicated average paths determined by four independent parameters. This largely increases the computation time. The solutions of Eqs. (28) keep, however, the following symmetry relations:

$$X_1(t;-\phi^0,u,-\alpha)=X_1(t;\phi^0,u,\alpha),$$

$$X_2(t;-\phi^0,u,-\alpha)=-X_2(t;\phi^0,u,\alpha). \tag{29}$$

When used in the integrals in Eqs. (26) these lead to

$$D_{12}(t)=D_{21}(t)=0,$$

$$D_{11}(t)=\frac{2\beta}{(2\pi)^{3/2}}\int\int_0^\infty d\phi^0\,du\,u^2\exp\left(-\frac{\phi^{02}+u^2}{2}\right)\times\int_0^{2\pi}d\alpha\cos\alpha X_1(t;\phi^0,u,\alpha),$$

$$D_{22}(t)=\frac{2\beta}{(2\pi)^{3/2}}\int\int_0^\infty d\phi^0\,du\,u^2\exp\left(-\frac{\phi^{02}+u^2}{2}\right)\times\int_0^{2\pi}d\alpha\sin\alpha X_2(t;\phi^0,u,\alpha). \tag{30}$$

Thus the diffusion tensor is diagonal and so is the LVC that is obtained from Eq. (30) by replacing X_i by $V_i^S(\mathbf{X}(t;S))$.

The trapped average trajectories do not contribute to the asymptotic diffusion coefficient nor to the large time LVC because they are incoherently mixed in the integrals. Only the opened average trajectories contribute at large time. Their asymptotic expressions, deduced from Eq. (28) and from the conservation of the average stream function (15), are $X_1(t;S)\cong C(S)+V_dt$ and $X_2(t;S)\cong-\phi^0/V_d$ for $t\gg\tau_f$. Introduced in Eqs. (30) they show that $D_{11}(t)$ has an asymptotic behavior linear in time $D_{11}(t)\to L_a(V_d)t$ and that $D_{22}(t)\to 0$. The constant $L_a(V_d)$ is the large time correlation of the velocity along V_d,

$$L_a(V_d)=\lim_{t\to\infty}L_{11}(t), \tag{31}$$

and since for the released trajectories $V_1(\mathbf{X}(t;S))\to V_d$, one can see from Eq. (25) that L_a can be written as

$$L_a(V_d)=\tilde{\mathcal{V}}^0(V_d)V_d, \tag{32}$$

where $\tilde{\mathcal{V}}^0(V_d)$ is the average of $\tilde{v}_1^0$ taken for the trajectories which are released by V_d. This quantity is not zero because the release of the trajectories appears especially when $\tilde{\mathbf{v}}^0$ is directed along $\mathbf{V}_d$ [$\alpha\in(-\pi/2,\pi/2)$]. An apparently paradoxical result is obtained, namely that particle trapping determines a superdiffusive transport.

Thus, a small average drift $\mathbf{V}_d$ produces the transition from the subdiffusive regime to a superdiffusive one in the direction of $\mathbf{V}_d$, while across $\mathbf{V}_d$ the process remains subdiffusive. The superdiffusion is due to a large time remnant correlation of the velocity determined essentially by the fact that the escaped trajectories ''remember'' the initial condition that situated them on paths which open to infinity.

The time evolution of the diffusion coefficients (30) is presented in Fig. 2(a) for $V_d=0.02$. Two dimensionless characteristic times can be noticed there: the flight time $\tau_f=1$ and the drift time $\tau_d=1/V_d$, which is the time necessary to traverse the correlation length with the average velocity. At small time t ($t<\tau_f$), the initial ballistic regime is observed $D_{ii}(t)\cong t$ and at $t>\tau_f$ trajectory trapping becomes effective and produces the decay of the diffusion. The diffusion is isotropic at this time $D_{11}(t)=D_{22}(t)$ and both are actually equal to the diffusion coefficient obtained for $V_d=0$ (dashed line). The effect of the average velocity appears for $t\gtrsim\tau_d$ and determines the anisotropy of the diffusion. The diffusion coefficient along $\mathbf{V}_d$ increases and eventually reaches the ballistic regime $D_{11}(t)\cong L_a t$. The cross $\mathbf{V}_d$ diffusion coefficient rapidly decays to zero [as $D_{22}(t)\sim t^{-3}$ in this case]. It is interesting to see how this picture evolves when V_d increases. When $V_d\simeq 1$, no trapping decay appears and the asymptotic ballistic regime is pushed at later times and a new transient regime of slowly increasing $D_{11}(t)$ appears [see Fig. 2(b) for $V_d=1$]. As V_d still increases, the slope of the transient regime goes to zero and its size extends to $t\to\infty$. Thus, at $V_d\gg 1$ a diffusive regime is obtained [see Fig. 2(c) for $V_d=10$].

The dispersion across $\mathbf{V}_d$ is subdiffusive for all values of the average velocity. The decay to zero of $D_{22}(t)$ depends on the large $|\mathbf{x}|$ tail of the EC of the stream function and it can be obtained analytically as

$$D_{22}(t)\sim\frac{f(V_d)}{V_d}|\mathcal{E}'(V_d t)|,\quad t\gg\tau_d \tag{33}$$

where $f(V_d)=\int d\phi^0\, du\, d\alpha \exp(-\phi^{02}/2-u^2/2)u^3\sin^2\alpha$, with the limits of integration depending on V_d, since the integral is performed on the domain of initial conditions corresponding to untrapped trajectories. For the EC (27), the asymptotic time dependence of the perpendicular diffusion coefficient is $D_{22}(t)\sim t^{-3}$ for all values of V_d. At large V_d, when trajectory trapping is negligible, $f(V_d)\to 1$.

The average Lagrangian velocity is obtained from Eq. (23) as

$$\langle v_1(\mathbf{x}(t))\rangle=\frac{2}{(2\pi)^{3/2}}\int\int_0^\infty d\phi^0\, du\, u \exp\left(-\frac{\phi^{02}+u^2}{2}\right)\times\int_0^{2\pi}d\alpha V_1(\mathbf{X}(t;S)) \tag{34}$$

and its time evolution is shown in Fig. 2(a). At small time $t<\tau_f$ the average Lagrangian velocity equals the Eulerian average V_d and at $t>\tau_f$ it decays to a smaller asymptotic value due to the trapping of a part of the trajectories [whose contributions vanish due to incoherent mixing in the integrals in Eq. (34)]. The fraction of the untrapped trajectories $n(V_d)$ can be obtained from the average Lagrangian velocity as

$$n(V_d)=\lim_{t\to\infty}\frac{\langle v_1(\mathbf{x}(t))\rangle}{V_d}. \tag{35}$$

When $V_d>1$, $n(V_d)\cong 1$ and $\langle v_1(\mathbf{x}(t))\rangle=V_d$.

The remnant correlation $L_a(V_d)$ and the number of untrapped trajectories $n(V_d)$ are plotted in Fig. 3. One can see that $n(V_d)\to 1$ at large V_d showing that all trajectories are released. $L_a(V_d)$ decays rapidly to zero when $V_d>1$ and thus the asymptotic ballistic term in the parallel diffusion disappears and the motion becomes diffusive at large V_d. At $V_d<1$, the asymptotic MSD is

$$\langle x_1^2(t)\rangle=[n^2(V_d)V_d^2+L_a(V_d)]t^2,\quad t\gg\tau_d \tag{36}$$

and the effective ballistic velocity appears as $V_b=\sqrt{n^2(V_d)V_d^2+L_a(V_d)}$ which is larger than V_d.

In conclusion, we can say that particle motion in a biased static stream function is rather complex and all the three types of evolution of the MSD appear: subdiffusion perpendicular to $\mathbf{V}_d$; superdiffusion and diffusion along $\mathbf{V}_d$, depending on the value of the average velocity V_d. The effective transport results from a competition between the trapping effect produced by the fluctuating part of the stochastic velocity field and the releasing effect determined by the average velocity.

IV. TIME-DEPENDENT TURBULENCE

We consider here the case of particle diffusion in a biased time-dependent stochastic stream function $\phi(\mathbf{x},t)$. The correlation time τ_c and the Kubo number K are finite. We use the same decorrelation trajectory method as in the static case.

The aim is to estimate the averages of the Lagrangian velocity in the subensembles (S) which are sufficient for calculating the LVC and the running diffusion coefficient. The difficulty comes from the fact that the average of the Lagrangian stream function is not known in the time-dependent problem: Eq. (18) is not valid in this case since the stream function is no longer conserved along the trajectory in each realization. However, the velocity is always tangent to the contour lines of $\phi(\mathbf{x},t)$ and only the explicit time variation of the stream function contributes to its variation along the trajectory,

$$d\phi(\mathbf{x}(t),t)/dt=\partial\phi(\mathbf{x}(t),t)/\partial t. \tag{37}$$

This equation can be used, in the particular case of the EC of $\phi(\mathbf{x},t)$ of the type (3), for determining the average of the Lagrangian stream function. The latter will be used as a constraint in determining the approximation for the average Lagrangian velocity in (S). The average Eulerian stream function $\Phi^S(\mathbf{x},t)$ in a subensemble (S) can be written according to Eq. (15) as

$$\Phi^S(\mathbf{x},t)=V_d x_2+\tilde{\Phi}^S(\mathbf{x})h(t), \tag{38}$$

where $\tilde{\Phi}^S(\mathbf{x})=\tilde{\phi}^0\mathcal{E}+\tilde{v}_i^{\,0}\mathcal{E}_{i\phi}$. In Lagrangian coordinates $\mathbf{x}(t)$, this quantity has the following general structure:

$$\Phi_L^S(t)\equiv\langle\phi(\mathbf{x}(t)),t\rangle_S=V_d\langle x_2(t)\rangle_S+G(t)h(t) \tag{39}$$

because the first term of Eq. (38) determines the first term in Eq. (39) and the time factor $h(t)$ propagates unchanged from Eulerian to Lagrangian coordinates. The $\mathbf{x}$ dependence of the stream function (i.e., the nonlinearity) generates the

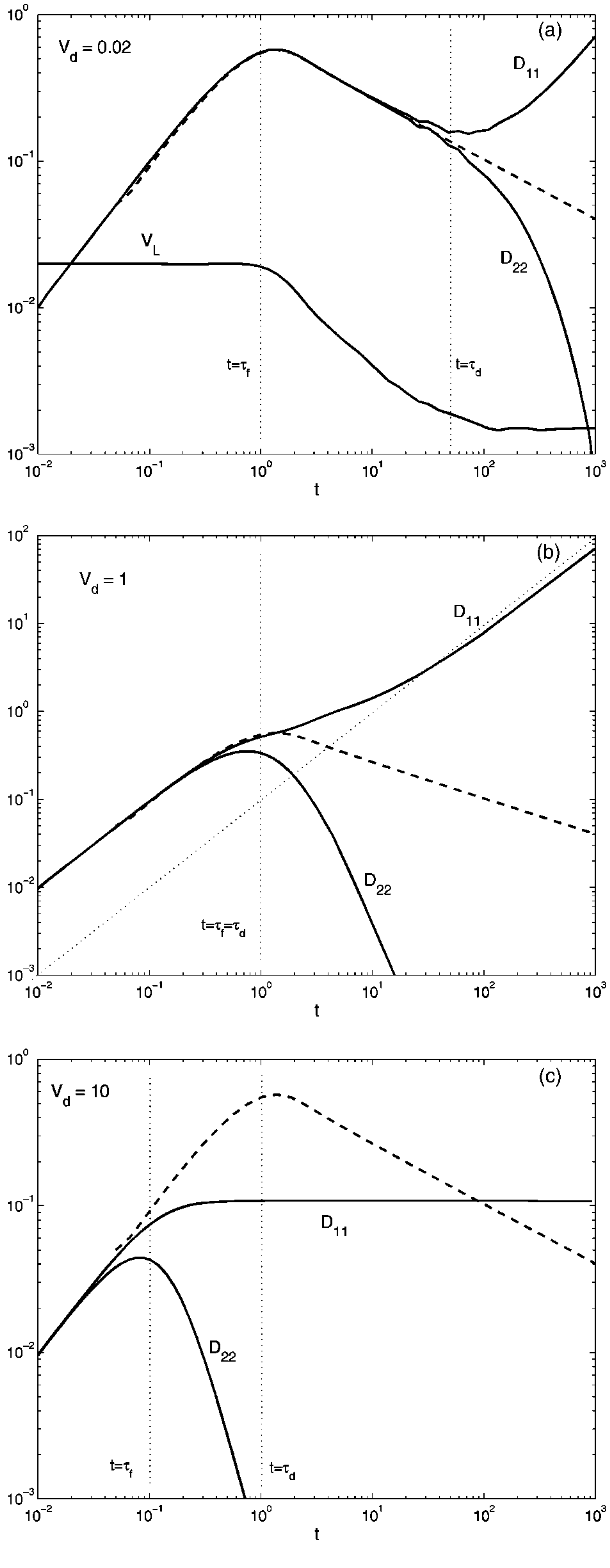

FIG. 2. (a) The time-dependent diffusion coefficients $D_{11}(t)$ and $D_{22}(t)$ normalized with β [Eqs. (30)] and the average Lagrangian velocity $V_L(t) \equiv \langle v_1(\mathbf{x}(t)) \rangle$ [Eq.(34)]. The diffusion coefficient for the driftless case (dashed line) is also represented for comparision. $V_d = 0.02$. (b) Same as in (a) for $V_d = 1$. (c) Same as in (a) for $V_d = 10$.

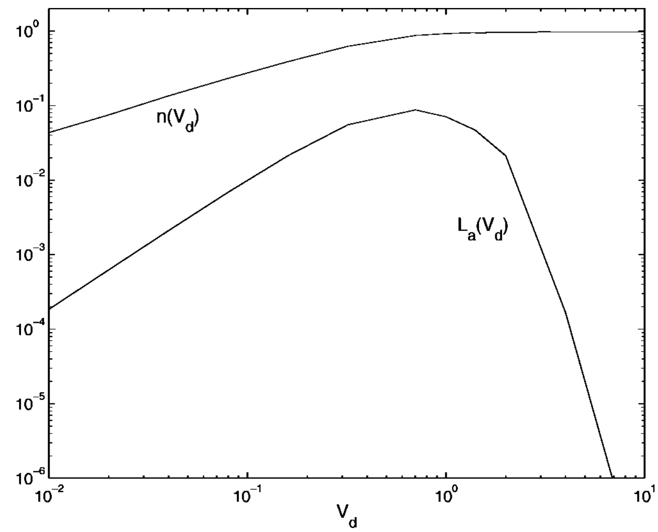

FIG. 3. The fraction of untrapped trajectories $n(V_d)$ and the remanent Lagrangian correlation $L_a(V_d)$ as functions of V_d.

other time function $G(t)$ [which depends also on V_d and on $h(t)$]. Averaging Eq. (37) in the subensemble and using Eq. (39) one obtains

$$\frac{d}{dt}\Phi_L^S(t) = \lim_{\delta t \to 0} \frac{\langle \phi(\mathbf{x}(t)), t + \delta t \rangle_S - \langle \phi(\mathbf{x}(t)), t \rangle_S}{\delta t} = G(t) h'(t). \tag{40}$$

Differentiating Eq. (39) and using Eq. (40), an equation for $G(t)$ is obtained,

$$V_d \frac{d}{dt} \langle x_2(t) \rangle_S + G'(t) h(t) = 0, \tag{41}$$

whose formal solution is

$$G(t) = \phi^0 - V_d \int_0^t d\tau \frac{1}{h(\tau)} \frac{d}{d\tau} \langle x_2(\tau) \rangle_S. \tag{42}$$

The subensemble average of the Lagrangian stream function can thus be represented as a functional of the average trajectory in (S),

$$\Phi_L^S(t) = \phi^0 h(t) - V_d \left(h(t) \int_0^t d\tau \frac{1}{h(\tau)} \frac{d}{d\tau} \langle x_2(\tau) \rangle_S - \langle x_2(t) \rangle_S \right). \tag{43}$$

We define, as in the static case, a deterministic trajectory $\mathbf{X}(t;S)$ so that the average of the Lagrangian stream function equals the average of the Eulerian ϕ calculated along this trajectory:

$$\Phi_L^S(t) = \Phi^S(\mathbf{X}(t;S), t). \tag{44}$$

As in the static case we approximate the average Lagrangian velocity by the Eulerian average calculated along the deterministic trajectory $\mathbf{X}(t;S)$,

$$\langle \mathbf{v}(\mathbf{x}(t),t)\rangle_S = \mathbf{V}^S(\mathbf{X}(t;S),t). \tag{45}$$

The average trajectory in the subensemble (S) can be determined by

$$\frac{d\langle \mathbf{x}(t)\rangle_S}{dt} = \mathbf{V}^S(\mathbf{X}(t;S),t). \tag{46}$$

We note that Eqs. (44), (45), and (46) are actually useful only if an equation for the deterministic trajectory $\mathbf{X}(t;S)$ can be deduced.

The average of the Lagrangian stream function (43) can be written using Eq. (46) as a functional of the deterministic trajectory $\mathbf{X}(t;S)$,

$$\Phi_L^S(t) = \phi^0 h(t) - V_d \int_0^t d\tau\, V_2^S(\mathbf{X}(\tau;S),\tau)\left(\frac{h(t)}{h(\tau)} - 1\right). \tag{47}$$

We show that the solution of Eq. (44) where $\Phi_L^S(t)$ is given by Eq. (47) can be obtained from the following time-dependent Hamiltonian system of equations with $\Phi^S(\mathbf{X},t)$ as the Hamiltonian function:

$$\frac{d\mathbf{X}(t;S)}{dt} = \left(\frac{\partial}{\partial X_2}, -\frac{\partial}{\partial X_1}\right)\Phi^S(\mathbf{X},t) \tag{48}$$

and initial condition $\mathbf{X}(0;S)=\mathbf{0}.$ Using Eq.(38), one can determine the variation of $\Phi^S(\mathbf{X},t)$ along the solution of Eq. (48) as

$$\frac{d\Phi^S(\mathbf{X}(t;S),t)}{dt} = [\Phi^S(\mathbf{X}(t;S),t) - V_d X_2(t;S)]\frac{h'(t)}{h(t)}$$

which integrated formally leads to

$$\Phi^S(\mathbf{X}(t;S),t) = \phi^0 h(t) - V_d \int_0^t d\tau\, V_2^S(\mathbf{X}(\tau;S),\tau)\left(\frac{h(t)}{h(\tau)} - 1\right). \tag{49}$$

Comparing Eqs. (47) and (49), we conclude that the solution of Eq. (48) ensures the equality of the two terms of Eq. (44) at any time. The deterministic trajectory $\mathbf{X}(t;S)$, the solution of Eq. (48), is thus an approximation of the average trajectory in the subensemble (S). We note that the essential condition in obtaining these results is the factorization of the time and space dependences in the EC of the stream function. For general time-dependent stream functions that do not have factorized EC, it was not possible to determine the average Lagrangian potential in (S). This method can still be used but without the insurance of taking into account the important constraint imposed by the stream function. The average Lagrangian velocity and the time dependent diffusion coefficients are calculated according to Eqs.(34) and (30), respectively, as in the static case.

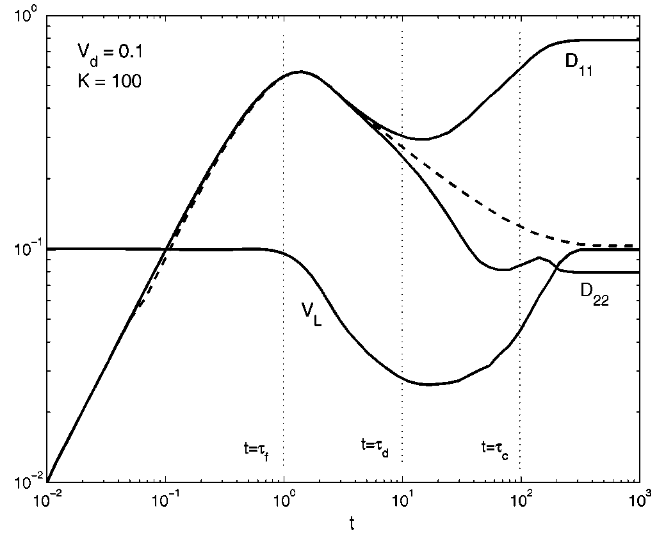

FIG. 4. Same as in Fig. 2(a) but for the time-dependent case $(K=100)$. The saturation of the diffusion coefficients can be observed.

The results are, however, rather different. Due to the time variation of the stream function, the average velocity in each subensemble goes asymptotically to $\mathbf{V}_d$ and the average trajectories obtained from Eq. (48) are all opened to infinity

$$X_1(t) \cong C_1(S) + V_d t, \quad t \gg K. \tag{50}$$

The trapping process still appears but it is only temporal: after performing a number of rotations [dependent on the five parameters in Eq. (48)], the initially trapped trajectories escape to infinity along $\mathbf{V}_d$. Consequently, $n(V_d)=1$ and $L_a(V_d)=0$ for arbitrary values of V_d. A finite asymptotic diffusion coefficient along $\mathbf{V}_d$ is obtained from Eq. (30). It is determined from the first term in Eq. (50), the second one exactly vanishes by integration over α. The perpendicular diffusion coefficient D_{22} saturates at a nonzero value in the time-dependent case. This can be seen in the asymptotic approximation of Eq. (48), which can be written as

$$\frac{dX_2}{dt} \cong -h(t)\frac{1}{V_d}\frac{\partial}{\partial t}\Delta\Phi^S(V_d t), \quad t \gg K.$$

The solution is $X_2(t) \cong -\Delta\Phi^S(V_d K)/V_d$ and it determines in Eq. (30) the following estimate for the asymptotic perpendicular diffusion coefficient using the EC (27):

$$D_{22} \sim f(V_d)\frac{K}{\left(1+\dfrac{K^2V_d^2}{2}\right)^2}, \quad K \gg \frac{1}{V_d} \tag{51}$$

where $f(V_d)$ is the integral defined after Eq. (33) and accounts for particle trapping. At large V_d, $f(V_d)\to 1$ and the quasilinear result (12) is recovered. Thus, the diffusion coefficient perpendicular to $\mathbf{V}_d$ is strongly dependent on the Eu-

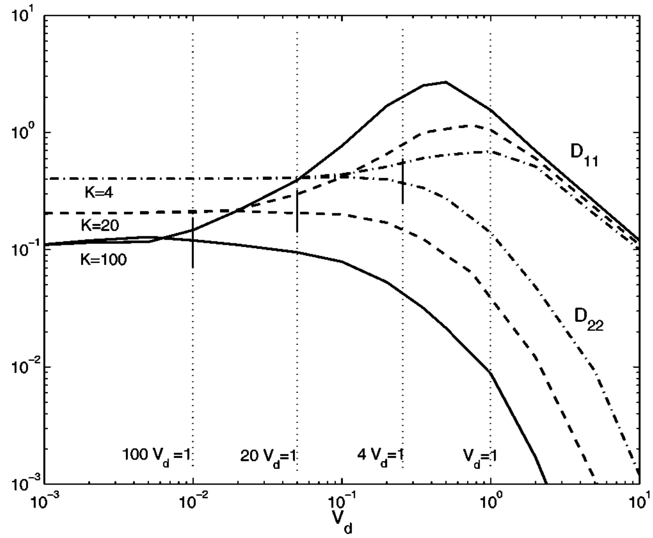

FIG. 5. The parallel, D_{11}, and the perpendicular, D_{22}, asymptotic diffusion coefficients as functions of V_d for $K=4$, 20, and 100.

lerian correlation of the stream function. The parallel diffusion coefficient also depends on the EC but the dependence is similar to that found in the driftless case in [11] and is rather weak.

An example of the time evolution of the diffusion coefficients is presented in Fig. 4 where the transitory regimes obtained in the static case [Fig. 2(a)] can be seen up to $t < \tau_c$. When $t \gtrsim \tau_c$ the saturation of the diffusion coefficients appears in both directions. The average Lagrangian velocity, also plotted in this figure, shows the presence of the temporal trapping of the trajectories: it is equal to V_d at $t \ll \tau_f$, then it decays due to trapping and at $t \gtrsim \tau_c$ it increases up to the Eulerian value due to the time variation of the stream function which releases all trajectories.

The asymptotic diffusion coefficients D_{11} and D_{22} are presented in Fig. 5 as functions of the average drift velocity V_d for various values of the Kubo number K. Three regimes can be identified. When $KV_d \ll 1$, the average velocity does not influence the diffusion and the transport is isotropic. At $V_d > 1$, the quasilinear result [Eqs. (11) and (12)] described in Sec. II is recovered. When there is no trajectory trapping ($K \ll 1$) only these two regimes appear. In the nonlinear case ($K > 1$) there is a third (''anomalous'') diffusion regime which is determined by the combined action of the average velocity and trajectory trapping. It appears when $V_d < 1$ and $K > 1$ so that $KV_d > 1$ (beyond the vertical dotted lines in Fig. 5) and consists of a very strong amplification of the parallel diffusion coefficient and a decrease of the perpendicular diffusion (see Fig. 5); the dependence of V_d of the latter can be approximated by Eq. (51) and is weaker than in the quasilinear case due to trajectory trapping [by the factor $f(V_d)$]. We note that in this regime an average velocity smaller than the stochastic one ($V_d < 1$) determines very important modifications of the diffusion coefficients while in the quasilinear case the effect appears only at drift velocities larger than the amplitude of the stochastic velocity ($V_d > 1$).

V. CONCLUSIONS

In conclusion, we have obtained here analytical expressions for the Lagrangian velocity correlations and for the time-dependent diffusion coefficients of particle motion in divergence-free stochastic velocity fields with constant average $\mathbf{V}_d$. These results have been obtained by the decorrelation trajectory method. It applies to Gaussian stochastic fields which are homogeneous and stationary. We consider both the static and the time-dependent problems.

For *static* stream functions, we have shown that the drift velocity $\mathbf{V}_d$ produces the transition from the subdiffusive process to a superdiffusive or diffusive transport, depending on the value of $\mathbf{V}_d$. In the absence of the average velocity ($V_d = 0$), all trajectories are trapped on the level lines of the stream function and particle motion is subdiffusive. A small average velocity ($V_d < V$) releases only a fraction of trajectories, $n(V_d)$; the other remaining trajectories are localized on small size contour lines of the stream function. A direct consequence of trajectory trapping is evidenced in the average Lagrangian velocity which is smaller than the Eulerian one by the factor $n(V_d)$. A more subtle and apparently paradoxical consequence of trapping is the superdiffusive, ballistic behavior of the MSD and of the parallel diffusion coefficient at large times. This is determined by the persistence of a long time correlation of the fluctuating Lagrangian velocity parallel to $\mathbf{V}_d$. We determine the fraction of untrapped trajectories n and the asymptotic Lagrangian velocity correlation L_a for arbitrary values of V_d. As V_d grows above V, $n(V_d) \to 1$ and $L_a(V_d)$ rapidly decays to zero, showing that all trajectories are released in the presence of such a large drift. Consequently, a diffusive regime is obtained at large V_d. The transport across $\mathbf{V}_d$ remains subdiffusive for all values of the drift velocity.

For *time-dependent* stream functions, the transport is diffusive both parallel and perpendicular to $\mathbf{V}_d$. We show that the average velocity can produce a very large amplification of the diffusion parallel to $\mathbf{V}_d$ and an important reduction of the perpendicular diffusion coefficient. The diffusion coefficients are determined for arbitrary values of $\mathbf{V}_d$ and of the Kubo number K. An ''anomalous'' regime is identified and is shown to correspond to the existence of trajectory trapping. It appears for average velocities smaller than the amplitude of the fluctuating part of the stochastic velocity ($V_d < V$) and at large Kubo numbers so that $KV_d/V > 1$.

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